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Penrose limit and NCYM theories in diverse dimensions

Somdatta Bhattacharya and Shibaji Roy

Saha Institute of Nuclear Physics
1/AF Bidhannagar, Calcutta-700 064, India
E-mails: som@theory.saha.ernet.in, roy@theory.saha.ernet.in

Abstract: We obtain the Penrose limit of NCYM theories in dimensions $3 \leq d \leq 6$ which originate from (D$(p-2)$, Dp) supergravity bound state configurations for $2 \leq p \leq 5$ in the so-called NCYM limit. In most cases the Penrose limit does not lead to solvable string theories except for six-dimensional NCYM theory. We obtain the masses of various bosonic coordinates and observe that they are light-cone time dependent and their squares can be negative as has also been observed in other cases in the literature. When the non-commutative effect is turned off we recover the results of Penrose limit of ordinary Dp-branes in the usual YM limit. We point out that for $d = 6$ NCYM theory, there exists another null geodesic in the neighborhood of which the Penrose limit leads to a solvable string theory. We briefly discuss the quantization of this theory and show that this pp-wave background is half supersymmetric.

Keywords: Superstrings and Heterotic Strings, D-branes, AdS-CFT and dS-CFT Correspondence, Penrose limit and pp-wave background.

Contents

1. Introduction

String theory in NSNS and RR pp-wave backgrounds has generated a lot of interest recently mainly because it has maximal supersymmetry for type IIB theory [1] (like flat Minkowski space and $AdS_5 \times S^5$), the corresponding GS action in the light-cone gauge is exactly solvable [2]–[4] and can be obtained by taking a Penrose limit [5][1] on a suitably chosen coordinates in the neighborhood of an appropriate null geodesic of $AdS_5 \times S^5$ [7]. Since the Maldacena conjecture [8]–[11] relates type IIB string theory on $AdS_5 \times S^5$ to the large N, $\mathcal{N} = 4$, $d = 4$, SU(N) gauge theory, taking a Penrose limit amounts to going to a particular subsector of the gauge theory [12]. In this subsector even though the 't Hooft coupling $g^2_{\rm YM} N$ is large as $N \to \infty$, there is an effective 't Hooft coupling $g^2_{\rm YM} N/J^2$ which remains fixed. Here J is the R-charge of a U(1) subgroup of the full SO(6) R-symmetry group of the $\mathcal{N} = 4$ gauge theory and scales as $J \sim \sqrt{N}$. As string theory in this case is exactly solvable, one uses this information to compute the anomalous dimensions of a particular set of operators in that subsector of gauge theory, corresponding to massive string modes. This is not possible for the $AdS_5 \times S^5$ background before taking the Penrose limit simply because the 't Hooft coupling is divergent. This has led BMN [12] to conjecture an exact correspondence between string theory in pp-wave background (beyond supergravity) and the subsector of gauge theory mentioned above and has been generalized to many other AdS/CFT-like examples.

While the near horizon geometry of D3-branes [8, 9] (which is $AdS_5 \times S^5$) is related to conformal field theory on the D3-brane world volume, the near horizon limits of other Dp-branes [13] are related to non-conformal theories on the world volume. It is of interest to see whether Penrose limit for the latter class of theories [14, 15] can teach us something new.

[1] The idea that any space-time obtained as solution of Einstein's equation can be reduced by Penrose limit to pp-wave background has been extended by Güven [6] for supergravity field equation including a p-form field strength.

In ref. [16], it has been pointed out that Penrose limit on the near horizon geometry of Dp-branes leads to time dependent pp-wave background of string theory. The string theories in these cases are not exactly solvable. However, for these classes of systems there is an intriguing connection between the associated time-dependent quantum mechanical problem and the RG flow in the dual gauge theory [16]–[18].[2] The Penrose limit of the near horizon geometry of NS5-branes, the supergravity dual of LST, is the Nappi-Witten background [20] and gives an exactly solvable string theory. This has been studied in [21]–[23]. Another class of non-local theories, namely, NCYM in (3+1) dimensions which originate from (D1, D3) bound state in the NCYM limit [24, 25] and its Penrose limit has also been studied in ref. [23]. However, four dimensional NCYM theory does not lead to a solvable string theory. Recently, the Penrose limit of OD5 theory [26] originating from (NS5, D5) bound state [27] in the so-called OD5 limit [28, 29] has been studied. Interestingly, it gives a solvable string theory. The string spectrum and their relation to the states in the six-dimensional "gauge" theory has been discussed in [30]. The Penrose limit of all ODp theories, four dimensional NCOS and OM theory has been discussed in [31].

In this paper we study the Penrose limit of NCYM theories [32, 33] (actually, their supergravity duals to be precise) in dimensions $3 \leq d \leq 6$. These theories in various dimensions arise from (D$(p-2)$, Dp) bound state (with $2 \leq p \leq 5$) of type II supergravities in the so-called NCYM limit. We obtain the Penrose limits for these supergravity configurations by making suitable coordinate changes in the neighborhoods of null geodesics following ref. [23]. A suitable scaling parameter in terms of number of Dp-branes, coupling constant and the noncommutativity parameter of the NCYM theories is chosen to obtain the Penrose limit. This gives a one parameter (l) family of string theories in a time dependent pp-wave background. However, because of the time dependence they are not solvable. For $p = 3$, we get the Penrose limit of four dimensional NCYM theory obtained in ref. [23], which reduces to the maximally supersymmetric Penrose limit of $AdS_5 \times S^5$ when the noncommutative effect is turned off. For general p we compute the mass^2's of the various bosonic fields and find that they can be negative (in both IR and UV) in some cases signalling a possible quantum mechanical instability for the corresponding string theory. When the noncommutativity parameter approaches zero (or in the IR) we get back the Penrose limit of ordinary Dp-branes obtained in ref. [16]. By examining the evolution equation along the geodesic we find that in six dimensions the Penrose limit of NCYM theory can give a solvable string theory when the parameter 'l' is saturated i.e. $l = 1$. In this case there exists a null geodesic very similar to the $AdS_5 \times S^5$ case and the Penrose limit gives a string theory where two of the eight bosonic coordinates have non-zero time independent masses. We briefly discuss the quantization of this theory and show that the background is half supersymmetric.

This paper is organized as follows. In section 2, we briefly review the (D$(p-2)$, Dp) supergravity bound state in the NCYM limit. The Penrose limit of the NCYM theories in various dimensions is discussed in section 3. In section 4, the (D5, D3) system and the

[2] See also [19] for some recent discussion on the possibility of obtaining solutions to a large class of string theories in pp-wave background with time dependent masses.

JHEP03(2003)053

Penrose limit of 6D NCYM theory is discussed separately which leads to a solvable string theory. Our conclusion is presented in section 5.

2. (D$(p-2)$, Dp) bound state and the NCYM limit

In this section we give a brief review of (D$(p-2)$, Dp) supergravity bound state configuration for $2 \leq p \leq 5$ and their NCYM limit.[3] These are the supergravity duals of NCYM theories in dimensions $3 \leq d \leq 6$. The string metric, the dilaton and the NSNS two-form potential have the forms,[4]

$$\begin{aligned}
ds^2 &= H^{1/2}\left[H^{-1}(-(d\tilde{x}^0)^2 + \sum_{i=1}^{p-2}(d\tilde{x}^i)^2) + H'^{-1}((d\tilde{x}^{p-1})^2 + (d\tilde{x}^p)^2) + d\tilde{r}^2 + \tilde{r}^2 d\Omega_{8-p}^2\right] \\
e^\phi &= g_s \frac{H^{(5-p)/4}}{H'^{1/2}} \\
B &= \tan\varphi H'^{-1} d\tilde{x}^{p-1} \wedge d\tilde{x}^p\,.
\end{aligned} \tag{2.1}$$

Here H and H' are two harmonic functions defined as

$$H = 1 + \frac{Q_p}{\tilde{r}^{7-p}}, \qquad H' = 1 + \frac{\cos^2\varphi Q_p}{\tilde{r}^{7-p}} \tag{2.2}$$

with $Q_p = g_s c_p \sqrt{n^2+m^2}\alpha'^{(7-p)/2}$, $c_p = 2^{5-p}\pi^{(5-p)/2}\Gamma((7-p)/2)$. Also g_s is the string coupling constant. n is the number of Dp-branes and m is the number of D$(p-2)$-branes per $(2\pi)^2\alpha'$ of two codimensional area of Dp-branes. $\tilde{r}$ is the radial coordinate transverse to Dp-branes and $d\Omega_{8-p}^2$ is the line element of unit $(8-p)$ dimensional sphere. The angle φ is defined as,

$$\cos\varphi = \frac{n}{\sqrt{n^2+m^2}}\,. \tag{2.3}$$

The charge Q_p above can therefore be written as $Q_p = (g_s c_p n \alpha'^{(7-p)/2})/\cos\varphi$ in terms of which the harmonic functions in (2.2) take the following forms,

$$H = 1 + \frac{n g_s c_p \alpha'^{(7-p)/2}}{\cos\varphi \tilde{r}^{7-p}}\,, \qquad H' = 1 + \frac{n g_s c_p \alpha'^{(7-p)/2}\cos\varphi}{\tilde{r}^{7-p}}\,. \tag{2.4}$$

The NCYM limit is defined by scaling $\alpha' \to 0$, keeping the following quantities fixed,

$$r = \frac{\tilde{r}}{\alpha'}\,, \qquad b = \frac{\alpha'}{\cos\varphi}\,, \qquad g = g_s \alpha'^{(p-5)/2}\,, \tag{2.5}$$

where r is the energy parameter and b is the noncommutativity parameter in the NCYM theory. The NCYM gauge coupling $\bar{g}_{\rm YM}^2$ is related to g and b as $\bar{g}_{\rm YM}^2 = (2\pi)^{p-2} g b$. Substituting (2.5) in (2.1) we get the NCYM supergravity configuration as follows,

$$ds^2 = \alpha' \frac{(ar)^{\frac{7-p}{2}}}{b}\left[-(dx^0)^2 + \sum_{i=1}^{p-2}(dx^i)^2 + \frac{1}{1+(ar)^{7-p}}((dx^{p-1})^2 + (dx^p)^2) + \right.$$

[3] For details see for example ref. [33].

[4] We do not give the RR-forms, as they are not important for our purpose here and can be found, for example, in [34]. See also [35].

$$
\left. + \frac{b^2}{(ar)^{7-p}}(dr^2 + r^2 d\Omega_{8-p}^2) \right]
$$

$$
\begin{aligned}
e^{\phi} &= g b^{\frac{5-p}{2}} \frac{(ar)^{\frac{(7-p)(p-3)}{4}}}{\sqrt{1+(ar)^{7-p}}} \\
B &= \frac{\alpha'}{b} \frac{(ar)^{7-p}}{1+(ar)^{7-p}} dx^{p-1} \wedge dx^p \,.
\end{aligned} \tag{2.6}
$$

In the above we have defined the fixed coordinates as,

$$
\begin{aligned}
x^{0,1,\ldots,p-2} &= \tilde{x}^{0,1,\ldots,p-2} \\
x^{p-1,p} &= \frac{b}{\alpha'} \tilde{x}^{p-1,p} \,.
\end{aligned} \tag{2.7}
$$

In terms of the fixed coordinates the noncommutativity parameter b is given by $[x^{p-1}, x^p] = ib$ and a is defined as $a^{7-p} = b/(nc_p g)$. The NCYM effective gauge coupling is $g_{\rm eff}^2 \sim \bar{g}_{\rm YM}^2 n r^{p-3}$. The supergravity description (2.6) is valid when the curvature in units of α' i.e. $\alpha' \mathcal{R} \sim g_{\rm eff}^{-1} \ll 1$ and the dilaton $e^{\phi} \ll 1$. The curvature condition implies

$$
g_{\rm eff}^2 \gg 1 \quad \Rightarrow \quad \begin{cases} r \ll (gbn)^{\frac{1}{3-p}} & \text{for} \quad p < 3 \\ r \gg (\frac{1}{gbn})^{\frac{1}{p-3}} & \text{for} \quad p > 3 \end{cases} \tag{2.8}
$$

and the dilaton condition implies

$$
e^{\phi} \ll 1 \quad \Rightarrow \quad g^2 b^{5-p} (ar)^{\frac{(7-p)(p-3)}{2}} \ll 1 + (ar)^{7-p} \,. \tag{2.9}
$$

In the UV, $ar \gg 1$ and this implies $e^{\phi} \sim (ar)^{(7-p)(p-5)/4}$ which vanishes for $p < 5$ and for $p = 5$, $e^{\phi} \sim g$ which will remain small if $g \ll 1$. In the IR when $ar \ll 1$, we note that the supergravity configuration reduces precisely to Dp-branes in the near horizon limit i.e. ordinary YM theory if we identfy,

$$
\frac{a^{\frac{7-p}{2}}}{b} = \frac{1}{\sqrt{g_{\rm YM}^2 c_p n}} = \frac{1}{\sqrt{gbc_p n}} \,, \tag{2.10}
$$

where $g_{\rm YM}^2$ is now the gauge coupling of ordinary YM theory. Note from (2.8) that for $p < 3$, NCYM theory is UV free, whereas for $p > 3$, the field theory breaks down and we need new degrees of freedom.

3. Penrose limit and NCYM theories

In order to take Penrose limit of the NCYM supergravity configuration (2.6), we first rescale the coordinates $x^{0,1,\ldots,p} \to (b/a)x^{0,1,\ldots,p}$. Then defining the scaling parameter

$$
R^2 = \alpha' \frac{b}{a^2} \tag{3.1}
$$

we rewrite the metric in (2.6) as,

$$ds^2 = R^2 e^{\frac{7-p}{2}U}\Bigg[-(dx^0)^2 + \sum_{i=1}^{p-2}(dx^i)^2 + \frac{1}{1+e^{(7-p)U}}((dx^{p-1})^2 + (dx^p)^2) + \\ + e^{-(5-p)U}\left(dU^2 + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\Omega_{6-p}^2\right)\Bigg]. \tag{3.2}$$

Here we have defined a new variable $e^U = (ar)$ and have written $d\Omega_{8-p}^2 = \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\Omega_{6-p}^2$. Now we consider a null geodesic in the (x^0, U, ψ) plane and so, we have for the geodesic $x^1 = x^2 = \cdots = x^p = 0$, $\theta = 0$. The effective lagrangian associated with this geodesic has the form,

$$\mathcal{L} = -e^{\frac{7-p}{2}U}((x^0)')^2 + e^{\frac{p-3}{2}U}(U')^2 + e^{\frac{p-3}{2}U}(\psi')^2\,, \tag{3.3}$$

where 'prime' denotes the derivative with respect to the affine parameter 'u' along the geodesic. Since the lagrangian does not depend on x^0 and ψ, we get the following constants of motion,

$$e^{\frac{7-p}{2}U}(x^0)' = E\,, \qquad e^{\frac{p-3}{2}U}\psi' = J\,. \tag{3.4}$$

For null geodesic, substituting (3.4) to (3.3) and equating it to zero, we get the evolution equation for U as,

$$U' = \frac{\sqrt{1 - e^{(5-p)U}l^2}}{e^U}\,, \tag{3.5}$$

where we have defined $J/E = l$ and also have scaled the affine parameter by E. From (3.5) we find that $l^2 \leq e^{(p-5)U}$. We remark that for $p = 5$ and $l = 1$, e^U = constant is a solution to the evolution equation. We will use this while discussing the Penrose limit and six dimensional NCYM theory. For future reference we note that the null geodesic in this case can be restricted to (x^0, ψ) plane. However, for $p \neq 5$, it is clear from (3.5) that such a null geodesic does not exist for any value of l. In other words, the null geodesic can not stay in the (x^0, ψ) plane even if we take $l = 1$ or 0. The parameter l has a clear geometric meaning, namely, it is the ratio of the angular momentum and the energy of a test particle moving along the trajectory described by the effective lagrangian (3.3). So, $l = 1$ means that for such a particle restricted to the trajectory the difference of energy and angular momentum must vanish. In the Penrose limit, when both the energy and the angular momentum are scaled as $R^2 \to \infty$, their difference remains finite and small in the neighborhood of the null geodesic. This fact gets translated to the gauge theory side as the difference of the energy and the U(1) R-charge of the states being finite and small at a fixed energy scale (since U is constant). An analogous situation also occurs for the maximally supersymmetric $AdS_5 \times S^5$ case, where the string theory results give information about the near BPS states in gauge theory.

Eq. (3.5) can be integrated to obtain u as,

$$u = \int \frac{e^U}{\sqrt{1 - e^{(5-p)U}l^2}}dU\,. \tag{3.6}$$

This relation can then be inverted to express e^U as a function of u and let us call that function as $e^U = f(u)$. Now we make the following change of variables from $(x^0,\, U,\, \psi) \to (u,\, v,\, x)$:

$$
\begin{aligned}
dU &= \frac{\sqrt{1-f^{5-p}l^2}}{f}du \\
dx^0 &= \frac{1}{f^{(7-p)/2}}du + 2dv + l\,dx \\
d\psi &= \frac{l}{f^{(p-3)/2}}du + dx\,.
\end{aligned}
\tag{3.7}
$$

Substituting (3.7) into (3.2) and rescaling the coordinates as, $u \to u$, $v \to v/R^2$, $\theta \to z/R$, $x \to x/R$, $x^{1,\ldots,p} \to x^{1,\ldots,p}/R$ and keeping others fixed along with $R \to \infty$, we find

$$
\begin{aligned}
ds^2 &= -4dudv - f^{\frac{3-p}{2}} l^2 \vec{z}^2 du^2 + f^{\frac{p-3}{2}}(1-l^2 f^{5-p})dx^2 + f^{\frac{7-p}{2}} \sum_{i=1}^{p-2} (dx^i)^2 + \\
&\quad + \frac{f^{\frac{7-p}{2}}}{1+f^{7-p}} \sum_{j=p-1}^{p} (dx^j)^2 + f^{\frac{p-3}{2}}(dz^2 + z^2 d\Omega^2_{6-p})\,,
\end{aligned}
\tag{3.8}
$$

where $\vec{z}^2 = z^2 = z_1^2 + \cdots + z_{7-p}^2$. This is the form of the metric in Rosen coordinates. It can be written in Brinkman form if we make another change of variables,

$$
\begin{aligned}
u &\to x^+ \\
x^{1,\ldots,p-2} &\to f^{\frac{p-7}{4}} x^{1,\ldots,p-2} \\
x^{p-1,p} &\to f^{\frac{p-7}{4}} \sqrt{1+f^{7-p}}\, x^{p-1,p} \\
x &\to \frac{f^{\frac{3-p}{4}}}{\sqrt{1-l^2 f^{5-p}}} x \\
\vec{z} &\to f^{\frac{3-p}{4}} \vec{z} \\
v &\to x^- - \frac{1}{8}\left[G_z \vec{z}^2 + G_{p-1,p} \sum_{j=p-1}^{p} (x^j)^2 + G_{1,\ldots,p-2} \sum_{i=1}^{p-2} (x^i)^2 + G_x x^2\right],
\end{aligned}
\tag{3.9}
$$

where

$$
\begin{aligned}
G_z &= \frac{\left(f^{\frac{p-3}{2}}\right)'}{f^{\frac{p-3}{2}}}\,, \qquad & G_{p-1,p} &= \frac{\left(\frac{f^{\frac{7-p}{2}}}{1+f^{7-p}}\right)'}{\left(\frac{f^{\frac{7-p}{2}}}{1+f^{7-p}}\right)} \\
G_{1,\ldots,p-2} &= \frac{\left(f^{\frac{7-p}{2}}\right)'}{f^{\frac{7-p}{2}}}\,, \qquad & G_x &= \frac{\left[f^{\frac{p-3}{2}}(1-l^2 f^{5-p})\right]'}{\left[f^{\frac{p-3}{2}}(1-l^2 f^{5-p})\right]}.
\end{aligned}
\tag{3.10}
$$

JHEP03(2003)053

Using (3.9), the metric in (3.8) takes the form,

$$
\begin{aligned}
ds^2 &= -4dx^+dx^- + \left[F_z\vec{z}^2 + F_{p-1,p}\sum_{j=p-1}^{p}(x^j)^2 + F_{1,\ldots,p-2}\sum_{i=1}^{p-2}(x^i)^2 + F_x x^2\right](dx^+)^2 + \\
&\quad + dx^2 + \sum_{k=1}^{p}(dx^k)^2 + d\vec{z}^2\,, \qquad (3.11)
\end{aligned}
$$

where

$$
\begin{aligned}
F_z &= -f^{3-p}l^2 + \frac{\left(f^{\frac{p-3}{4}}\right)''}{f^{\frac{p-3}{4}}}\,, \qquad F_{p-1,p} = \frac{\left(\frac{f^{\frac{7-p}{4}}}{\sqrt{1+f^{7-p}}}\right)''}{\left(\frac{f^{\frac{7-p}{4}}}{\sqrt{1+f^{7-p}}}\right)} \\
F_{1,\ldots,p-2} &= \frac{\left(f^{\frac{7-p}{4}}\right)''}{f^{\frac{7-p}{4}}}\,, \qquad F_x = \frac{\left[f^{\frac{p-3}{4}}\sqrt{1-l^2f^{5-p}}\right]''}{\left[f^{\frac{p-3}{4}}\sqrt{1-l^2f^{5-p}}\right]}\,. \qquad (3.12)
\end{aligned}
$$

Here 'prime' denotes derivative with respect to $u = x^+$. Eq. (3.11) is the NCYM supergravity metric in the Penrose limit in Brinkman form. The forms of the dilaton and the NSNS two form in these coordinate system is given as,

$$
\begin{aligned}
e^{\phi} &= gb^{\frac{5-p}{2}}\frac{f^{\frac{(7-p)(p-3)}{4}}}{\sqrt{1+f^{7-p}}} \\
B &= f^{\frac{7-p}{2}}dx^{p-1}\wedge dx^p\,. \qquad (3.13)
\end{aligned}
$$

The mass^2's of various bosonic coordinates are given as $m_z^2 = -F_z$, $m_{p-1,p}^2 = -F_{p-1,p}$, $m_{1,\ldots,p-2}^2 = -F_{1,\ldots,p-2}$ and $m_x^2 = -F_x$ and they can be calculated from (3.12) as,

$$
\begin{aligned}
m_z^2 &= \frac{7-p}{16}f^{3-p}\left[-(3-p)f^{p-5} + (p+1)l^2\right] \\
m_{1,\ldots,p-2}^2 &= \frac{7-p}{16}f^{3-p}\left[-(3-p)f^{p-5} + (13-3p)l^2\right] \\
m_{p-1,p}^2 &= \frac{(7-p)f^{3-p}}{16(1+f^{7-p})^2}\Big[-(3-p)f^{p-5}(1+f^{7-p})^2 + (13-3p)l^2(1+f^{7-p})^2 + \\
&\qquad + 4(19-3p)f^2(1-f^{5-p}l^2)(1+f^{7-p}) - 4(5-p)f^{7-p}l^2\times \\
&\qquad \times(1+f^{7-p}) - 12(7-p)f^{9-p}(1-f^{5-p}l^2)\Big] \\
m_x^2 &= m_{1,\ldots,p-2}^2\,, \qquad (3.14)
\end{aligned}
$$

where we have used from (3.5)

$$
f' = \sqrt{1-f^{5-p}l^2}\,, \qquad f'' = -\frac{5-p}{2}l^2f^{4-p}\,, \qquad f''' = -\frac{5-p}{2}(4-p)l^2f^{3-p}f'\,. \qquad (3.15)
$$

Several comments are in order here. First of all, note that the metric is light-cone time (x^+) depenedent and therefore does not lead to solvable string theories. Secondly, we comment that the scaling parameter defined by $R^2 = \alpha' b/a^2$, with $a^{7-p} = b/(nc_p g)$, is taken to infinity while taking the Penrose limit and in terms of the parameters of the theory this means that it can be achieved either by (i) $b \to \infty$, a = fixed, or by (ii) $a \to 0$, b = fixed. From the relation between a and b given above the condition (i) implies $g \to 0$, $n \to \infty$ with bg = fixed and b^2/n = fixed. On the other hand the condition (ii) simply implies $n \to \infty$ and b, g fixed. For case (i) we have large noncommutativity and for case (ii) since $a \to 0$ means $ar \to 0$,[5] therefore, we will not have any noncommutative effect. However, we would like to emphasize that the limit for case (ii) as mentioned above is not quite right and we will be misled if we look at only the metric and the dilaton field. Actually, a closer look at the other gauge fields reveal that we will not get back the commutative theory by taking this naive limit and the proper limit along with $a \to 0$ and $n \to \infty$ should be $b \to 0$, $g \to \infty$ with bg = fixed and $a^{(7-p)/2}/b \sim 1/\sqrt{n} \to 0$.

Let us discuss case (ii) first and then we will discuss case (i). For case (ii) as we mentioned in section 2, the NCYM supergravity configuration would reduce to ordinary YM theory and we also have to use the relation (2.10). The Penrose limit of NCYM theory also reduces to that of ordinary Dp-branes (in the near horizon limit). Indeed we note that since $f^{7-p} \ll 1$ in this case, the functions $F_{p-1,p} = F_{1,\ldots,p-2} = (f^{(7-p)/4})''/f^{(7-p)/4}$ and masses satisfy

$$m_{p-1,p}^2 = m_{1,\ldots,p-2}^2 = \frac{7-p}{16} f^{3-p}\left[-(3-p)f^{p-5} + (13-3p)l^2\right]. \tag{3.16}$$

The forms of the other masses remain the same as given in (3.14). However, note that these formulae are not the same as given in [16, eq. (5.19)] for ordinary Dp-branes, because, here we have defined $ar = e^U = f(x^+)$. The parameter 'a' does not have any obvious meaning in the commutative theory. But we note that this parameter can be completely absorbed by scaling $f \to f/a$, $l^2 \to a^{5-p}l^2$, $x^+ \to x^+/a$ and $x^- \to ax^-$. In that case the metric (3.11) as well as the masses would take very similar form as given in [16, (5.18) and (5.19)] and the discussions of mass^2 would also be the same. Unlike in that reference our formula is valid even for the $p = 5$ case.

For case (i) we have noncommutativity since a = fixed. We will look at the theory in the UV where $ar \gg 1$. In this region the behavior of mass^2 would be

$$m_{p-1,p}^2 = \frac{7-p}{16} f^{3-p}\left[-(11-p)f^{p-5} + (p+1)l^2\right] \tag{3.17}$$

with all other mass^2's remaining the same as given in (3.14). Also the behavior of the dilaton is

$$e^{\phi} \sim \bar{g}_{\mathrm{YM}}^2 b^{\frac{3-p}{2}} f^{\frac{(7-p)(p-5)}{4}}. \tag{3.18}$$

We note from (3.17) and (3.18) that for $p \geq 3$, all the mass^2's are always positive except $m_{p-1,p}^2$ and $e^{\phi} \ll 1$. Since for the consistency of geodesic motion (3.5) we find,

$$f \leq \left(\frac{1}{l^2}\right)^{\frac{1}{5-p}}, \qquad \text{for} \quad p < 5$$
$$\text{and} \quad l^2 \leq 1, \qquad \text{for} \quad p = 5 \tag{3.19}$$

[5]This can also be achieved by going to the IR, where the theory would become commutative again.

we conclude that there is no region of f where $m^2_{p-1,p}$ can become positive for $p < 5$. However for $p = 5$, it remains negative for all $l^2 < 1$, except for $l^2 = 1$, where $m^2_{4,5} = 0$. We will return to the last case in the next section.

In case of ordinary Dp-branes, it was shown in [16] that when the mass^2 becomes negative, the supergravity description should be given either by the S-dual one (for p odd) or by lifting it to M-theory (for p even) by an RG flow where mass^2 becomes positive again. However, this does not seem to be possible in the present case, since e^ϕ would blow up by going to the dual frame. For $p = 2$ all the mass^2's become negative and so, we conclude that the mass^2 becoming negative is a generic feature of all NCYM theories in the Penrose limit in dimensions $3 \leq d \leq 6$ and needs a more careful study to understand their physical meaning. However, for $d = 6$ there is a special case where such problem does not arise and we will consider this case in the next section.

We would like to emphasize that it would be too naive to conclude that the appearance of scalars of negative mass^2 on the world-sheet would necessarily imply the instability of the background space-time. As has been already pointed out in [16] that these tachyons are world-sheet tachyons and they arise in Brinkman coordinates only in the light-cone gauge (in the static gauge the pp-wave gives rise to the world-sheet theory of massless but interacting scalars). For time independent masses the issue of stability of the pp-wave background has been studied in the presence of negative mass^2 in [36, 37]. By looking at the geodesic equation of the transverse coordinates from the pp-wave metric in this case, it is easy to see that a test particle moving along the geodesic will experience a repulsive tidal force and will be pushed off to infinity with time. This means that for strings the center of mass (or zero mode) will experience a similar force in this background. However, this phenomenon by itself does not imply instability if the strings move rigidly along the geodesic. But, it can be seen that when $m_i P_- > 1/l_s^2$, where m_i is the mass of the i-th coordinate, P_- is the light-cone momentum and l_s is the string length scale, the different parts of the string experience different repulsive tidal forces causing the string to rip apart. Morever, under the same condition, it can be seen that truly stringy unstable modes appear, although they are finite in number. In other words, the appearance of unstable modes imply a string theoretic instability of the background. However, it has been argued in [37], that by imposing an infrared cut-off on the coordinate x^i, the background can be made stable. A careful analysis in [36] made at the level of linearized field equations showed that the fluctuations around the pp-wave background with negative but constant mass^2 are indeed stable. For the case of time dependent mass^2 i.e. for the case we have discussed so far, it is in general difficult to solve the geodesic equations, but for the cases where they can be solved [19], it would be interesting to understand the issue of stability along the lines [36, 37].

Finally, we note that for $p = 3$, the mass^2 relations given in (3.14) simplify to

$$\begin{aligned} m_z^2 &= m_1^2 = m_x^2 = l^2 \\ m_{2,3}^2 &= \frac{l^2}{(l^4 + \sin^4(lu))^2} \left[l^8 - \sin^8(lu) - 2\sin^2(lu)\cos^2(lu)(\sin^4(lu) - 5l^4) \right], \end{aligned} \tag{3.20}$$

where we have integrated the evolution equation (3.5) and used the form of f obtained from there as,

$$f(u) = e^U = \frac{1}{l}\sin(lu)\,. \tag{3.21}$$

By scaling $u = x^+ \to ax^+$, $x^- \to x^-/a$ and $l \to l/a$, the metric (3.11) reduces to

$$\begin{aligned} ds^2 \;=&\; -4dx^+dx^- - l^2\left(\vec{z}^2 + (x^1)^2 + x^2 + g(x^+)((x^2)^2 + (x^3)^2)\right)(dx^+)^2 + \\ &+ dx^2 + \sum_{i=1}^{3}(dx^i)^2 + d\vec{z}^2\,, \end{aligned} \tag{3.22}$$

where

$$g(x^+) = \frac{1}{(l^4 + a^4\sin^4(lx^+))^2}\left[l^8 - a^8\sin^8(lx^+) - \frac{1}{2}a^4\sin^2(2lx^+)(a^4\sin^4(lx^+) - 5l^4)\right]. \tag{3.23}$$

This is exactly the same form of the metric obtained in ref. [23]. For $a \to 0$, $g(x^+) \to 1$ and we get back the maximally supersymmetric pp-wave resulting from $AdS_5 \times S^5$.

4. Penrose limit, 6d NCYM and a solvable string theory

In this section we will discuss the Penrose limit of (D5, D3) bound state system in the NCYM limit (this is the supergravity dual of 6d NCYM theory) separately and will see that the Penrose limit in this case would lead to a solvable string theory. The full supergravity configuration of (D3, D5) system is given as [33, 34],

$$\begin{aligned} ds^2 &= H^{1/2}\left[H^{-1}\left(-(d\tilde{x}^0)^2 + \sum_{i=1}^{3}(d\tilde{x}^i)^2\right) + H'^{-1}((d\tilde{x}^4)^2 + (d\tilde{x}^5)^2) + d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2\right] \\ e^\phi &= g_s H'^{-1/2} \\ B &= \tan\varphi H'^{-1} d\tilde{x}^4 \wedge d\tilde{x}^5 \\ A^{(2)} &= n\alpha' \sin^2\theta d\psi \wedge d\phi \\ A^{(4)} &= -\frac{1}{2}m\alpha' H'^{-1}\sin^2\theta d\tilde{x}^4 \wedge d\tilde{x}^5 \wedge d\psi \wedge d\phi + \frac{\sin\varphi}{g_s}H^{-1}d\tilde{x}^0 \wedge d\tilde{x}^1 \wedge d\tilde{x}^2 \wedge d\tilde{x}^3\,, \end{aligned} \tag{4.1}$$

where we have written $d\Omega_3^2 = \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\phi^2$. The harmonic functions H and H' and the angle $\cos\varphi$ are defined in (2.2) and (2.3) with $p = 5$. The NCYM limit is given in (2.5) with $p = 5$. The supergravity dual of 6d NCYM theory would then be given as,

$$\begin{aligned} ds^2 &= \alpha'\frac{ar}{b}\left[-(dx^0)^2 + \sum_{i=1}^{3}(dx^i)^2 + \frac{1}{1+(ar)^2}\sum_{j=4}^{5}(dx^j)^2 + \frac{b^2}{a^2}\left(\frac{dr^2}{r^2} + d\Omega_3^2\right)\right] \\ e^\phi &= g\frac{ar}{\sqrt{1+(ar)^2}} \\ B &= \frac{\alpha'}{b}\frac{(ar)^2}{1+(ar)^2}dx^4 \wedge dx^5 \\ A^{(2)} &= n\alpha'\sin^2\theta d\psi \wedge d\phi \\ A^{(4)} &= -\frac{1}{2}n\frac{(\alpha')^2}{b}\frac{(ar)^2}{1+(ar)^2}\sin^2\theta dx^4 \wedge dx^5 \wedge d\psi \wedge d\phi + \frac{(\alpha')^2}{gb^2}(ar)^2 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\,. \end{aligned} \tag{4.2}$$

Here the fixed coordinates $x^{0,\dots,5}$ are as defined before and the NCYM coupling is $\bar{g}^2_{\rm YM} = (2\pi)^3 gb$. The parameter $a^2 = b/(ng)$, n being the number of D5-branes. The Penrose limit of this theory for $l^2 < 1$ is discussed in section 3 and is taken by scaling the coordinates $x^{0,\dots,5} \to (b/a)x^{0,\dots,5}$. We have also defined the scaling parameter $R^2 = (\alpha' b)/a^2$, where now $b/a = \sqrt{bng} = \sqrt{(\bar{g}^2_{\rm YM} n)/(2\pi)^3}$. The configuration is given in Brinkman coordinates as,

$$
\begin{aligned}
ds^2 &= -4dx^+dx^- - \left(m_z^2 \vec{z}^2 + m_x^2\left(x^2 + \sum_{i=1}^{3}(x^i)^2\right) + m_{4,5}^2\left((x^4)^2+(x^5)^2\right)\right)(dx^+)^2 + \\
&\quad + dx^2 + d\vec{z}^2 + \sum_{i=1}^{5}(dx^i)^2 \\
e^\phi &= g\frac{f}{\sqrt{1+f^2}} \\
B &= f dx^4 \wedge dx^5 \\
A^{(2)} &= \frac{l}{gf} z^2 dx^+ \wedge d\phi \\
dA^{(4)} &= \frac{l}{g} dx^4 \wedge dx^5 \wedge dx^+ \wedge dz_1 \wedge dz_2\,,
\end{aligned} \tag{4.3}
$$

where mass^2's are as given in (3.14) with $p = 5$. Also note that $z^2 = \vec{z}^2 = z_1^2 + z_2^2$, where $z_1 = z\cos\phi$, $z_2 = z\sin\phi$ and therefore, $d(z^2 d\phi) = 2dz_1 \wedge dz_2$. We have defined as before $ar = e^U = f(x^+)$, where f is obtained by solving the evolution equation (3.6) and has the form $f(x^+) = \sqrt{1-l^2}x^+$. Thus (4.3) gives the pp-wave limit of the supergravity dual of $6d$ NCYM theory. As we mentioned earlier in the UV where $ar \gg 1$, even though the dilaton in (4.3) $e^\phi \ll 1$, $m_{4,5}^2$ remains negative in this case. But all other mass^2's namely m_x^2, m_z^2 are positive. Also since the masses are light-cone time dependent, it does not lead to solvable string theory for $l^2 < 1$. When $ar \ll 1$, (4.3) would reduce to the Penrose limit of D5-brane (in the near horizon limit). Note that in this case the parameter 'a' has to be absorbed by proper scaling as mentioned earlier in section 3.

Now looking at the evolution equation (3.5) for the general value of the parameter l^2, we notice that there exists a null geodesic corresponding to $l^2 = 1$ for which e^U or U remains constant. The null geodesic is now confined in the (x^0, ψ) plane as in the case of maximally supersymmetric $AdS_5 \times S^5$. The geodesic is $U = U_0 =$ constant, $x^{1,\dots,5} = \theta = 0$, $x^0 = \psi = \lambda$, where λ is related to the affine parameter. Now let us look at the metric (3.2) for $p = 5$ and make the following coordinate change,

$$
\begin{aligned}
U &\to U_0 + e^{-U_0/2}x \\
\theta &\to e^{-U_0/2}z \\
x^{1,2,3} &\to e^{-U_0/2}x^{1,2,3} \\
x^{4,5} &\to e^{-U_0/2}\left(1+e^{2U_0}\right)^{1/2}x^{4,5} \\
x^0 &\to e^{-U_0/2}\left(x^+ + x^-\right) \\
\psi &\to e^{-U_0/2}\left(x^+ - x^-\right).
\end{aligned} \tag{4.4}
$$

By further rescaling the coordinates as $x^+ \to x^+$, $x^- \to x^-/R^2$, $x \to x/R$, $z \to z/R$, $x^{1,\ldots,5} \to x^{1,\ldots,5}/R$ and taking $R \to \infty$, the metric takes the form,

$$ds^2 = -4dx^+dx^- - \vec{z}^2(dx^+)^2 + \sum_{i=1}^{5}(dx^i)^2 + dx^2 + d\vec{z}^2\,, \tag{4.5}$$

where in writing the above metric we have rescaled $x^\pm \to e^{\pm U_0/2}x^\pm$. It is clear from (4.5) that only two of the eight bosonic coordinates are massive and time independent. So, this will lead to a solvable string theory. The masses of the bosonic fields can be made arbitrary by scaling $x^\pm \to \mu^{\pm 1}x^\pm$ and then the metric as well as the other fields would take the following forms,

$$\begin{aligned}
ds^2 &= -4dx^+dx^- - \mu^2\vec{z}^2(dx^+)^2 + d\vec{z}^2 + \sum_{i=3}^{8}(dz_i)^2 \\
e^\phi &= g\frac{e^{U_0}}{\sqrt{1+e^{2U_0}}} \\
B &= -e^{U_0}dz_3\wedge dz_4 \qquad \Rightarrow \qquad dB = 0 \\
F^{(3)} &= dA^{(2)} = -\frac{2}{g}e^{-U_0}\mu dx^+\wedge dz_1\wedge dz_2 \\
F^{(5)} &= dA^{(4)} - \frac{1}{2}B\wedge F^{(3)} \\
&= \frac{2\mu}{g}(dx^+\wedge dz_1\wedge dz_2\wedge dz_3\wedge dz_4 + dx^+\wedge dz_5\wedge dz_6\wedge dz_7\wedge dz_8)\,.
\end{aligned} \tag{4.6}$$

Note that we have renamed the coordinates $(x^5,\, x^4,\, x^3,\, x^2,\, x^1,\, x)$ as $(z_3,\, z_4,\, z_5,\, z_6,\, z_7,\, z_8)$. It would be interesting to study the complete quantization of the above system, but in the following we will quantize only the bosonic closed string sector and mention a few words about the fermionic sector for completeness. Also, we mention that for case (ii) discussed in the previous section, the above configuration matches with the Penrose limit of D5-branes considered in [30].

The bosonic part of the Green-Schwarz action is

$$-4\pi\alpha' S_b = \int d\tau\int_0^{2\pi\alpha' p^+} d\sigma(\eta^{ab}G_{\mu\nu}\partial_a x^\mu\partial_b x^\nu + \epsilon^{ab}B_{\mu\nu}\partial_a x^\mu\partial_b x^\nu)\,, \tag{4.7}$$

where $\eta^{ab} = \mathrm{diag}(-1,\,1)$ is the world-sheet metric and $\epsilon^{\tau\sigma} = 1$. For the background (4.6), the $B_{\mu\nu}$ term will not contribute and so, we get,

$$-4\pi\alpha' S_b = \int d\tau\int_0^{2\pi\alpha' p^+} d\sigma(\eta^{ab}\partial_a z_i\partial_b z_i + \mu^2 z_k^2)\,, \tag{4.8}$$

where we have used the light-cone gauge $x^+ = \tau$ and in the above $i = 1,\ldots,8$ and $k = 1,2$. The equations of motion following from (4.8) are

$$\begin{aligned}
\eta^{ab}\partial_a\partial_b z_l &= 0\,, \qquad \text{for} \quad l = 3,\ldots,8 \\
\eta^{ab}\partial_a\partial_b z_k - \mu^2 z_k &= 0\,, \qquad \text{for} \quad k = 1,2\,.
\end{aligned} \tag{4.9}$$

The equations of motion can be solved as usual by Fourier expanding the bosonic fields z_i, with $i = 1, \ldots, 8$. The bosonic part of the light-cone hamiltonian would be given as,

$$2p^- = \sum_n \left(N_n^{(l)} \frac{|n|}{\alpha' p^+} + N_n^{(k)} \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} \right), \tag{4.10}$$

where $k = 1, 2$ corresponds to the massive bosons and $l = 3, \ldots, 8$ corresponds to the massless bosons. Now in order to relate the string spectrum to the states in NCYM theory, we write,

$$\frac{\partial}{\partial x^+} = \frac{\partial}{\partial x^0} + \frac{\partial}{\partial \psi}, \qquad \frac{\partial}{\partial x^-} = \frac{e^{-U_0}}{R^2} (\frac{\partial}{\partial x^0} - \frac{\partial}{\partial \psi}). \tag{4.11}$$

In terms of the generators of translation of original x^0 (before rescaling by $(b/a)x^0$) we find

$$\begin{aligned} \frac{2p^-}{\mu} &= i\frac{\partial}{\partial x^+} = \frac{b}{a}E - J_V \\ 2\mu p^+ &= i\frac{\partial}{\partial x^-} = \frac{e^{-U_0}}{R^2} \left(\frac{b}{a}E + J_V \right), \end{aligned} \tag{4.12}$$

where we have used $i\frac{\partial}{\partial x^0} = \frac{b}{a}E$ and $-i\frac{\partial}{\partial \psi} = J_V$. We thus find a correspondence of string propagation in the background (4.6) with the states in $6d$ NCYM theory. To be precise, the string spectrum given in (4.10) corresponds to states in NCYM theory with energy and U(1) R-charge

$$\frac{b}{a}E, \qquad J_V \sim \frac{b}{a^2} \to \infty, \qquad \text{with} \quad \frac{b}{a}E - J_V = \text{finite}. \tag{4.13}$$

We note from (4.12) that for $R^2 \to \infty$,

$$R^2 \mu p^+ = e^{-U_0} J_V. \tag{4.14}$$

The light-cone energy therefore takes the form,

$$\frac{2p^-}{\mu} = \sum_n \left(N_n^{(l)} e^{U_0} \frac{b}{J_V a^2} |n| + N_n^{(k)} \sqrt{1 + e^{2U_0} \frac{b^2}{a^4 J_V^2} n^2} \right). \tag{4.15}$$

We would like to mention that the NCYM supergravity configuration (4.3) before taking the Penrose limit has $SO(4) \simeq SU(2)_L \times SU(2)_R$ isometry of S^3 which is the full R-symmetry group of the $6d$ NCYM theory. The $U(1)_1 \times U(1)_2 \equiv U(1)_V \times U(1)_A$ subgroup of this full isometry group corresponds to the isometry of ψ and ϕ in $d\Omega_3^2$. If we write the two bosonic fields z_1 and z_2 in terms of complex scalars $z = (z_1 + iz_2)/2$ and $\bar{z} = (z_1 - iz_2)/2$, then they will carry $U(1)_2 = U(1)_A$ charge corresponding to the angular coordinate ϕ. On the other hand the massless scalars $z_3, \ldots, z_8$ are $U(1)_A$-charge neutral, where $U(1)_{V(A)}$ denotes the vector(axial vector) subgroup of $U(1)_L \times U(1)_R$. We thus find that the states with non-zero $U(1)_A$ charge have different energies than those with vanishing $U(1)_A$-charge. A similar conclusion has been drawn in [30] for the case of $6d$ "gauge" theory, the holographic dual of (NS5, D5) bound state (in the OD5 limit), although the details are quite different.

The fermionic part of the Green-Schwarz action comes from the direct covariantization of the quadratic fermionic terms of the flat space action [38] and has the form [4]

$$-4\pi\alpha' S_f = i\int d\tau \int_0^{2\pi\alpha' p^+} d\sigma (\eta^{ab}\delta_{IJ} - \epsilon^{ab}\rho_{3IJ})\partial_a x^\mu \theta^I \Gamma_\mu (\mathcal{D}_b\theta)^J \,, \tag{4.16}$$

where the supercovariant derivative is defined as,

$$\begin{aligned}\mathcal{D}_b = \partial_b + \frac{1}{4}\partial_b x^\nu \Bigg[&\left(\omega_{\hat\lambda\hat\rho\nu} - \frac{1}{2}H_{\hat\lambda\hat\rho\nu}\rho_3\right)\Gamma^{\hat\lambda\hat\rho} - \\ &- e^\phi \left(\frac{1}{3!}F_{\hat\lambda\hat\rho\hat\sigma}\Gamma^{\hat\lambda\hat\rho\hat\sigma}\rho_1 + \frac{1}{2(5!)}F_{\hat\mu_1\ldots\hat\mu_5}\Gamma^{\hat\mu_1\ldots\hat\mu_5}\rho_0\right)\Gamma_\nu\Bigg]\end{aligned} \tag{4.17}$$

In the above $I, J = 1, 2$ are the labels of two real MW spinors. The hatted indices are the tangent space indices. ρ's in I, J space are related to the Pauli matrices as $\rho_3 =$ diag $(1, -1)$, $\rho_1 = \sigma_1$ and $\rho_0 = i\sigma_2$. $\Gamma^{\hat\mu}$'s are 32×32 Dirac matrices satisfying the Clifford algebra

$$\{\Gamma^{\hat\mu}, \Gamma^{\hat\nu}\} = 2\eta^{\hat\mu\hat\nu} \,, \tag{4.18}$$

where $\eta^{\hat\mu\hat\nu}$ is the mostly positive lorentzian metric and $\Gamma^{\hat\mu_1\cdots}$ is as usual the totally antisymmetric product of gamma matrices. In the light-cone gauge we set $x^+ = \tau$ and $\Gamma^+\theta^I = 0$.[6] In this gauge the non-zero contribution to the action comes only when both the 'internal' and the 'external' $\partial_a x^\mu$ factor of the action become $\delta^\mu_+\delta^0_a$. For the background (4.6) (with a rescaling $x^- \to -1/2x^-$) we can calculate

$$\omega_{\hat\lambda\hat\rho\mu}\Gamma^{\hat\lambda\hat\rho} = -2\mu^2(z_1\Gamma^{+1} + z_2\Gamma^{+2}) \tag{4.19}$$

and so the supercovariant derivative simplifies to

$$\begin{aligned}\mathcal{D}_0 &= D_0 - \frac{1}{4}e^\phi\left(F_{+12}\Gamma^{+12}\rho_1 + \frac{1}{2}F_{+1234}(\Gamma^{+1234} + \Gamma^{+5678})\rho_0\right)\Gamma_+ \\ \mathcal{D}_1 &= \partial_1 \,,\end{aligned} \tag{4.20}$$

where $D_0 = \partial_0 - \frac{1}{2}\mu^2(z_1\Gamma^{+1} + z_2\Gamma^{+2})$. So, the action would take the form,

$$\begin{aligned}-4\pi\alpha' S_f = i\int d\tau\int_0^{2\pi\alpha' p^+} d\sigma \Bigg[&-\delta_{IJ}\theta^I\Gamma_+\Bigg(D_0 - \frac{1}{4}\Bigg(F_{+12}\Gamma^{+12}\rho_1 + \frac{1}{2}F_{+1234}\times \\ &\times(\Gamma^{+1234}+\Gamma^{+5678})\rho_0\Bigg)\Gamma_+\Bigg)^{JK}\theta_K - \\ &- \rho_{3IJ}\theta^I\Gamma_+\partial_1^{JK}\theta_K\Bigg] \,.\end{aligned} \tag{4.21}$$

[6] After rescaling $x^- \to -1/2x^-$ in the background (4.6) it is easy to verify that for the upper $+$ and lower $-$ indices, it is not necessary to distinguish between the hatted and the unhatted indices. The same is true for the transverse indices $i = 1, \ldots, 8$. So, in the following all the indices are hatted except the lower $+$ and upper $-$ indices and we will remove the hats while writing the indices explicitly.

JHEP03(2003)053

One can write down the equations of motion from the above action and quantize the system in a straightforward way. We will not study the quantization of the fermionic sector in any further detail here. However, we will show that pp-wave background of the $6d$ NCYM theory given in (4.6) preserves only half of the space-time supersymmetry.

Now in order to study the supersymmetry of the above background we follow closely the analysis made in ref. [1]. We first write the supersymmetry variations of the dilatino and the gravitino as follows [39],

$$\delta\chi = \left[\Gamma^\mu\partial_\mu\phi - \frac{1}{4(3!)}e^\phi F_{\hat\lambda\hat\rho\hat\nu}\Gamma^{\hat\lambda\hat\rho\hat\nu}\rho_1\right]\epsilon \tag{4.22}$$

$$\begin{aligned}\delta\psi_\mu &= \mathcal{D}_\mu\epsilon \equiv \Bigg[\partial_\mu + \frac{1}{4}(\omega_{\hat\lambda\hat\rho\mu} - \frac{1}{2}H_{\hat\lambda\hat\rho\mu}\rho_3)\Gamma^{\hat\lambda\hat\rho} - \\ &\qquad - \frac{e^\phi}{4}\left(\frac{1}{3!}F_{\hat\lambda\hat\rho\hat\sigma}\Gamma^{\hat\lambda\hat\rho\hat\sigma}\rho_1 + \frac{1}{2(5!)}F_{\hat\mu_1\ldots\hat\mu_5}\Gamma^{\hat\mu_1\ldots\hat\mu_5}\rho_0\right)\Gamma_\mu\Bigg]\epsilon\,,\end{aligned} \tag{4.23}$$

where $\epsilon = \binom{\epsilon_1}{\epsilon_2}$ is a two component spinor on which the matrices ρ_0 and ρ_1 act and satisfy the chirality condition $\Gamma_{11}\epsilon_{1,2} \equiv \Gamma^{+-}\Gamma^1\ldots\Gamma^8\epsilon_{1,2} = -\epsilon_{1,2}$. Since the background is bosonic, for consistency we set $\delta\chi = \delta\psi_\mu = 0$. So, from (4.22) we get,

$$\left[\Gamma^\mu\partial_\mu\phi - \Gamma^{+12}\Lambda\right]\epsilon = 0\,, \tag{4.24}$$

where we have defined $\Lambda = (e^\phi/4)F_{+12}\rho_1 = \lambda\rho_1$. Note that for the background (4.6) the dilaton is constant and therefore the above equation can be satisfied if $\Gamma^{+12}\epsilon = 0$. Furthermore, from (4.23) we obtain,

$$\left[\partial_\mu + \frac{1}{4}\omega_{\hat\lambda\hat\rho\mu}\Gamma^{\hat\lambda\hat\rho} - \frac{e^\phi}{4}\left(\frac{1}{3!}F_{\hat\lambda\hat\rho\hat\sigma}\Gamma^{\hat\lambda\hat\rho\hat\sigma}\rho_1 + \frac{1}{2(5!)}F_{\hat\mu_1\ldots\hat\mu_5}\Gamma^{\hat\mu_1\ldots\hat\mu_5}\rho_0\right)\Gamma_\mu\right]\epsilon = 0\,. \tag{4.25}$$

In components it gives $\partial_-\epsilon = 0$ and so, ϵ is independent of x^- coordinate. For the i-th component ($i = 1,\ldots,8$), we get

$$(\partial_i + \Omega_i)\epsilon = 0\,, \tag{4.26}$$

where,

$$\Omega_i = \begin{cases}\Gamma^+\Gamma^i(I\Lambda + J\Lambda')\,, & \text{for} \quad i = 1,2\\ \Gamma^+\Gamma^i(-I\Lambda + J\Lambda')\,, & \text{for} \quad i = 3,4\\ \Gamma^+\Gamma^i(-I\Lambda + K\Lambda')\,, & \text{for} \quad i = 5,\ldots,8\,.\end{cases} \tag{4.27}$$

Here $I = \Gamma^{12}$, $J = \Gamma^{1234}$, $K = \Gamma^{5678}$, $\Lambda' = (e^\phi/4)F_{+1234}\rho_0 = \lambda'\rho_0$. We also note the identities, $I^2 = -1$, $J^2 = K^2 = 1$, $IJ = JI$, $IK = KI$, $JK = KJ$ and $\Lambda^2 = \lambda^2$ $\Lambda'^2 = -\lambda'^2$, $\Lambda\Lambda' - \Lambda'\Lambda = -2\lambda\lambda'\rho_3$. Now since, $\Omega_i\Omega_j = 0$ for all i,j, so we get from (4.26)

$$\epsilon = \left(1 - \sum_{i=1}^{8} z_i\Omega_i\right)\psi\,, \tag{4.28}$$

where the spinor ψ depends only on x^+ coordinate. Finally, the $+$ component of the killing spinor equation (4.25) gives,

$$\left[\partial_+ - \frac{\mu^2}{2}\sum_{i=1}^{2} z_i\Gamma^+\Gamma^i - (2I\Lambda + J\Lambda' + K\Lambda')\right]\epsilon = 0\,. \tag{4.29}$$

JHEP03(2003)053

Substituting (4.28) in (4.29) and using the identities mentioned above eq. (4.29) can be simplified in the following form

$$\begin{aligned}\partial_+\psi - (2I\Lambda + (J+K)\Lambda')\psi = & \sum_{i=1}^{2} z_i \left(\frac{\mu^2}{2} - 4\lambda^2 - 2\lambda'^2 + 2IK\lambda\lambda'\rho_3\right)\Gamma^i\Gamma^+\psi + \\ & + \sum_{i=3}^{4} 2z_i(I(K-2J)\lambda\lambda'\rho_3 - \lambda'^2)\Gamma^i\Gamma^+\psi + \\ & + \sum_{i=5}^{8} 2z_i(I(J-2K)\lambda\lambda'\rho_3 - \lambda'^2)\Gamma^i\Gamma^+\psi\,. \end{aligned} \tag{4.30}$$

Since the lhs of (4.30) is independent of z_i coordinates whereas the rhs depends on z_i, they must be separately zero. It is clear that the rhs of (4.30) can vanish only if $\Gamma^+\psi = 0$ i.e. only half of the components of ψ are non-trivial indicating that the background is half supersymmetric. This is also consistent with dilatino equation (4.24). The exact form of the killing spinor ϵ can be obtained by putting the lhs of (4.30) to zero, solving this for ψ and substituting it in (4.28). This therefore concludes our discussion of Penrose limit of $6d$ NCYM theory.

5. Conclusion

In this paper we have studied the Penrose limit of the supergravity duals of NCYM theories in dimensions $3 \leq d \leq 6$. The supergravity descriptions are obtained from (D$(p-2)$, Dp) bound state configurations for $2 \leq p \leq 5$ of type II string theories in the so-called NCYM limit. We found that the Penrose limit in these cases gives a one parameter (l) family of string theories in a time dependent pp-wave background. Because of the time dependence the corresponding string theories are not exactly solvable. We also obtained the expressions of mass^2's of various bosonic fields. We found that for $p \geq 3$, all the mass^2's remain positive except the ones associated with the bosonic fields corresponding to the noncommutative coordinates. The mass^2's for the latter coordinates are always negative. However, for $p = 2$, not only the mass^2's of the bosonic fields of the noncommutative directions, but also those associated with the commuting directions are negative. So, we conclude that mass^2 becoming negative is a generic feature of the NCYM theories in the various dimensions we studied and requires a careful study to understand their physical meaning. In particular, it has been pointed out in [16], that for such time dependent pp-wave background there is a map bewteen the associated time dependent quantum mechanical problem and the RG flow in the dual gauge theory. It should be interesting to understand the precise meaning of this map in the present context.

In obtaining the Penrose limit we have defined a scaling parameter $R^2 = \alpha' b/a^2 \to \infty$, in terms of the known parameters of the NCYM theories. $R^2 \to \infty$ means either (i) $b \to \infty$, $a =$ fixed, or (ii) $a \to 0$, $b \to 0$, $n \to \infty$, $g \to \infty$ with bg fixed and $a^{(7-p)/2}/b \sim 1/\sqrt{n} \to 0$, where b is the noncommutativity parameter, n is the number of Dp-branes and $bg \sim \bar{g}^2_{\mathrm{YM}}$, is the gauge coupling. For case (i) there is a large noncommutativity and all our discussions above apply to this case. For case (ii) the theories become commutative. We have shown how for case (i) our results reproduce the Penrose limit of four dimensional NCYM theory studied in [23]. We have also shown that for case (ii) we recover the Penrose limit of ordinary Dp-branes (in the YM limit) studied in [16].

All the discussions above apply to the NCYM theories in $3 \leq d \leq 6$ for the generic value of the parameter l. We found that in $d = 6$, there exists another Penrose limit of NCYM theory, when the parameter l is saturated i.e. $l = 1$, which leads to a solvable string theory. We have studied the quantization of this string theory in the complete NSNS and RR pp-wave background. We studied the quantization of the bosonic sector in detail and pointed out some features of the fermionic sector where we have also shown that the pp-wave background is half supersymmetric. We have obtained the spectrum of the light-cone hamiltonian of the string theory and mentioned their relations to the states in 6d NCYM theory.

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JHEP03(2003)053